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1984 J. Phys. A: Math. Gen. 17 2531

(http://iopscience.iop.org/0305-4470/17/12/023)

J. Phys. A: Math. Gen. 17 (1984) 2531–2534. Printed in Great Britain

Critical relaxation of the three-state Potts model

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Received 20 December 1983, in final form 8 March 1984

Abstract. The dynamical exponent z is obtained through a Monte Carlo renormalisation group calculation for the two-dimensional three-state Potts model with nonconserved dynamics. The value of z is found to be $z = 2.7 \mp 0.4$.

1. Introduction

Recent work on the critical properties of the two-dimensional three-state Potts model include calculations of static exponents (see for example Rebbi and Swendsen (1980)), and specific heat and free energy (Novotny 1982). The static exponents are also known exactly (Baxter 1980, 1982, Alexander 1975).

However, less is known about the dynamics of this model. Forgacs *et al* (1980) calculated the dynamical critical exponent z for nonconserved dynamics as $z = 2.25$ using a Migdal-type recursion method. Later Tobochnik and Jayaprakash (1982) predicted a value of $z = 2.7 \mp 0.4$ through a Monte Carlo renormalisation group method.

In this paper we present a Monte Carlo renormalisation group calculation of the dynamical critical exponent for the two-dimensional three-state Potts model on a square lattice with nonconserved dynamics, by a method introduced by N Jan and D Stauffer (1982, unpublished) and later modified by Aydın and Yalabık (1984). Our estimate of z is 2.7 ∓ 0.4 also.

In § 2, a brief explanation about the method and the procedure is given. Results are presented and discussed in § 3.

2. Method and calculations

The two-dimensional three-state Potts model can be defined through the Hamiltonian H

$$-\frac{H}{kT} = K \sum_{ij} \delta_{s_i, s_j} \qquad s_i = 1, 2, 3 \tag{1}$$

where k is the Boltzmann constant, T is the temperature and $\{s_i\}$ represent spin variables on a lattice. K corresponds to the nearest-neighbour coupling and has a critical value K_c, $K_c = \ln(1 + \sqrt{3})$ which is an exact result (Potts 1952).

The Monte Carlo renormalisation group method used in this work involves the calculation of z from the relaxation of the order parameter and the renormalised order parameter using dynamical scaling. The details of the method used in computation

0305-4470/84/122531 + 04$02.25

were discussed in an earlier paper (Aydın and Yalabık 1984), and only the main features will be discussed here.

Computations were carried out on a 256×256 lattice. The system was relaxed from a configuration in which all spins are in the $s = 1$ state. Standard Monte Carlo procedure is used to find the new configuration corresponding to the development of the system in time. The order parameters of the original and renormalised lattices were computed after every Monte Carlo step per spin as an average of eleven independent runs. Values of the projection in the $s = 1$ direction of the original and renormalised order parameters m and m' are expected to obey the relation

$$m(t - t_0) \sim m'[b^z(t - t_0')] \tag{2}$$

where t is the time, b is the renormalisation scale change, t_0 and t_0' are the possible shifts in time scales of m and m', introduced to obtain better matching. The values of t_0 and t_0' can be determined using the relation

$$m(t - t_0) = b^{-\beta/\nu} m'(t - t_0'). \tag{3}$$

Equation (3) can be obtained easily from (2) by using the asymptotic relation $m \sim t^{-\beta/\nu z}$ (Suzuki 1977). β/ν is known to be equal to 1/7.5 (Baxter 1980, 1982, Alexander 1975). Using a fitting procedure for (3), t_0 and t_0' are calculated from m and m' for a certain time interval. Then z is estimated through another fitting procedure using (2).

The values of z for different time intervals are given in table 1. The choice of the best time interval requires a careful analysis of the order parameter data. Fluctuations in the order parameter values may result in very large errors in the estimate of z if the time interval is not chosen carefully. To demonstrate the validity of scaling, we have plotted m and rescaled values of m' in figures 1 and 2 using (3) and (2) respectively.

Table 1. The values of z obtained from various time intervals. The invervals are listed in decreasing order in the goodness of fit. The last two intervals correspond to the two extreme values that can be obtained for z.

Interval (MC steps)	350–600	400–500	400–550	340–600	360–600	320–600	300–450	440–600
z	2.736	2.77	2.71	2.85	2.63	2.99	3.36	2.06

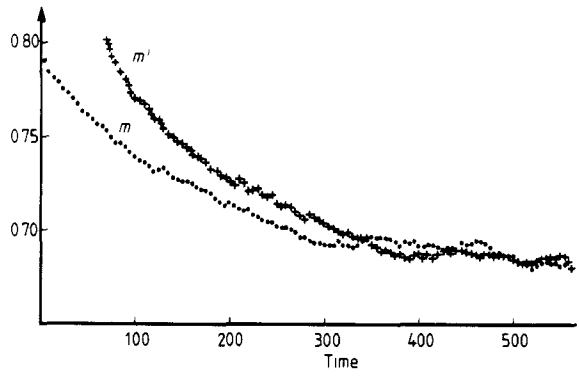

Figure 1. The variation of m and $b^{-\beta/\nu}m'$ as a function of $t - t_0$ and $t - t_0'$ respectively for the best match.

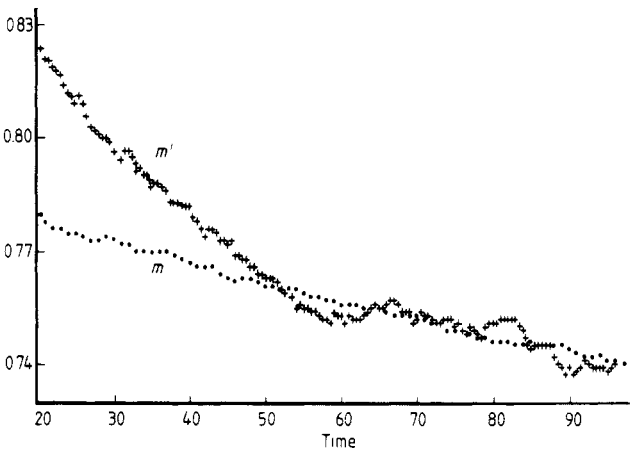

Figure 2. The variation of m and m' as a function of $t - t_0$ and $b^{-z}(t - t_0)$ respectively for the best match.

Scaling is quite good as can be seen from the figures. The best fit to the data results in a z value of $z = 2.74$. The standard deviation corresponding to this estimate of z (obtained from the eleven independent runs) is 0.4. On the other hand, another estimate of z (based on a number of intervals that give good fits) results in the value $z = 2.7$ and a corresponding standard deviation of 0.4.

3. Conclusions

In the present work, the value of the dynamical critical exponent z is calculated for the two-dimensional three-state Potts model on a square lattice with nonconserved dynamics. The value we have found for z, $z = 2.7 \mp 0.4$ is consistent with the earlier estimate of Tobochnik and Jayaprakash (1982). The accuracy of the method can be increased if the calculations are performed on a larger lattice and the number of independent runs is increased. Within the accuracy of our work, it is unlikely that the model we have studied is in the same dynamical universality class as the two-dimensional Ising model.

Acknowledgments

The authors would like to thank the Middle East Technical University Computing Center for their cooperation.

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